Graphing Reciprocal Functions

These notes are intended as a supplement of chapter 4 in your workbook.

Reciprocal Functions

A reciprocal function is any function of the form:

$$y = \frac{1}{f(x)}$$

It is a special transformation that changes the *y*-coordinates of points on $f(x)$ to their reciprocal. In other words,

$$(x, y) \to \left(x, \frac{1}{y}\right)$$

Although this is the only thing that happens, it can result in some very interesting graphs. This lesson will explain some steps to make graphing reciprocal functions easier.

Graphing Reciprocal Functions

1. Draw the graph of $f(x)$ first.

2. On a separate graph, draw vertical asymptotes wherever $f(x)$ has *x*-intercepts.

   Explanation: An *x*-intercept is a point where the *y*-coordinate is zero, such as $(3,0)$. If you take the reciprocal of the *y*-coordinate, you have a fraction with zero in the denominator.

   $$(3,0) \to \left(3, \frac{1}{0}\right)$$

   Since we can't divide by zero, there will be a non-permissible value (a vertical asymptote) at that *x*-value on our reciprocal graph.

3. Draw a horizontal asymptote at $y = 0$ if your original graph's *y*-coordinates go towards positive or negative infinity.

   Explanation: As the *y*-coordinates of $f(x)$ get larger, the *y*-coordinates of the reciprocal graph will get smaller. Specifically, they will get closer and closer to 0 but will never reach 0. This is asymptotic behavior, indicating the presence of a horizontal asymptote at $y = 0$.

4. Plot all the points on your original graph where the y-coordinate is equal to 1 or –1. These points will also be on the reciprocal graph and are called **invariant points**.

   Explanation: Any point whose y-coordinate is 1 or –1 will not change when you apply the reciprocal transformation. For example:

$$(3,\ 1) \rightarrow \left(3,\ \frac{1}{1}\right) \rightarrow (3,\ 1)$$

$$(5,\ -1) \rightarrow \left(5,\ \frac{1}{-1}\right) \rightarrow (5,\ -1)$$

5. For all points on $f(x)$ where the y-coordinate is a local maximum or local minimum, determine the corresponding point on the reciprocal graph.

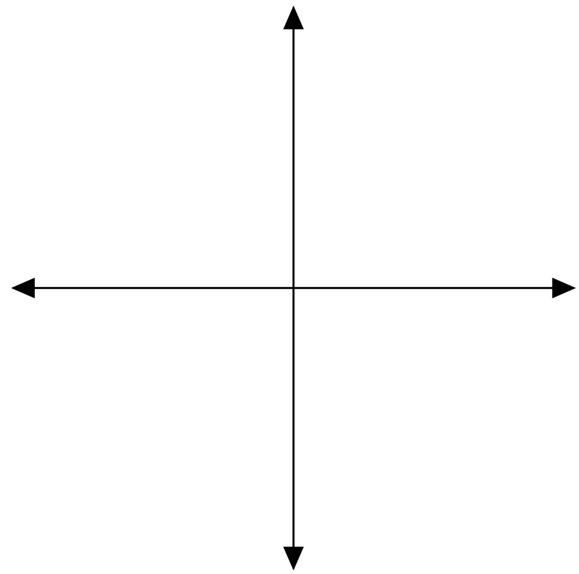
   Explanation: Local maximums and local minimums are points on your graph where the direction changes. Local maximums on $f(x)$ will become local minimums on the reciprocal graph (and vice versa). For example:

   If $(3,\ 6)$ is a local maximum on $f(x)$, then $\left(3,\ \frac{1}{6}\right)$ will be a local minimum on $y = \frac{1}{f(x)}$.

6. Draw your graph, showing asymptotic behavior as appropriate.

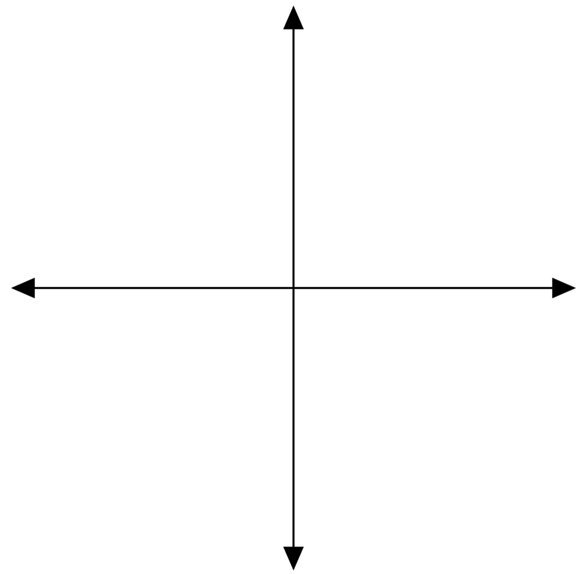
Example 1

Graph $y = \dfrac{1}{x^2 - 4}$



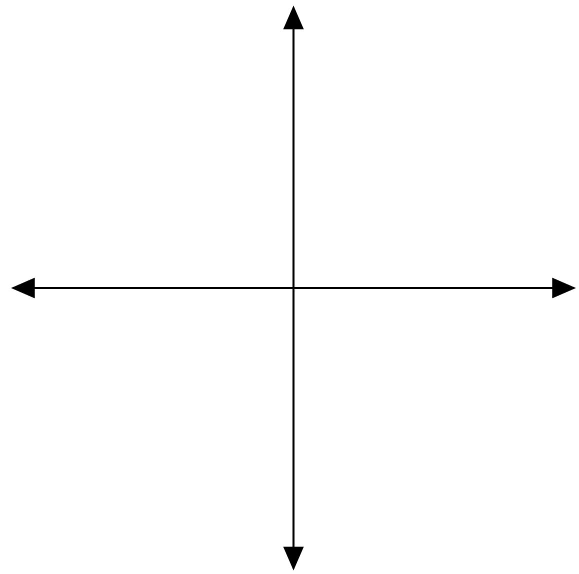
Example 2

Graph $y = \dfrac{1}{x^2 + 1}$



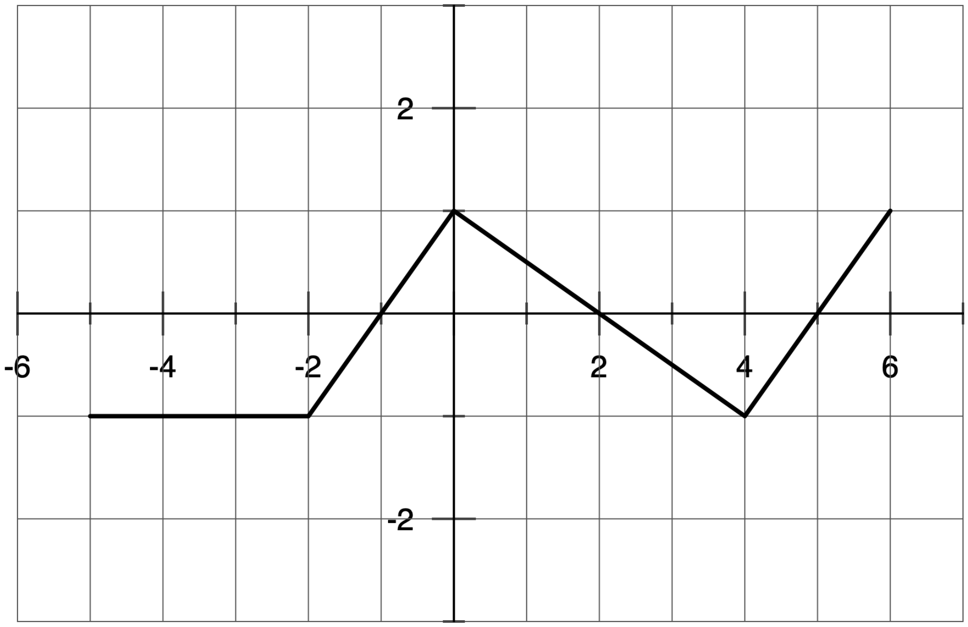
Example 3

Graph $y = \dfrac{1}{\sqrt{x+4}}$



Example 4

Given $f(x)$ below, draw $y = \dfrac{1}{f(x)}$.



Homework: Supplemental Worksheet #12

Supplemental Worksheet #12

Graph each of the following reciprocal functions:

1. $y = \dfrac{1}{x^2 - 9}$

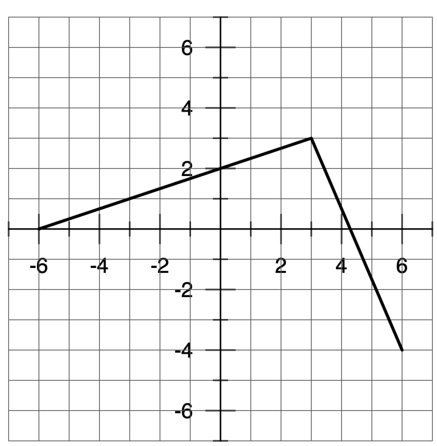
2. $y = \dfrac{1}{x^2 + 2}$

3. $y = \dfrac{1}{(x-4)^3}$

4. $y = \dfrac{1}{x - 3}$

5. $y = \dfrac{1}{\sqrt{x - 2}}$

For each graph of $f(x)$ below, sketch a graph of $y = \dfrac{1}{f(x)}$.

6. 

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